Kuhn, Michael; Prettner, Klaus

Working Paper

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GLO Discussion Paper, No. 462

Provided in Cooperation with:
Global Labor Organization (GLO)

Rising longevity, increasing the retirement age, and the consequences for knowledge-based long-run growth*

Michael Kuhn and Klaus Prettner

a) Wittgenstein Centre (Univ. Vienna, IIASA, VID/ÖAW),
Vienna Institute of Demography
Welthandelsplatz 2, Level 2
A-1020, Vienna, Austria
email: michael.kuhn@oeaw.ac.at

b) University of Hohenheim
Institute of Economics
Schloss, Osthof-West,
70593 Stuttgart, Germany
email: klaus.prettner@uni-hohenheim.de
Corresponding author

Abstract

We assess the long-run growth effects of rising longevity and increasing the retirement age when growth is driven by purposeful research and development. In contrast to economies in which growth depends on learning-by-doing spillovers, raising the retirement age fosters economic growth. How economic growth changes in response to rising life expectancy depends on the retirement response. Employing numerical analysis we find that the requirement for experiencing a growth stimulus from rising longevity is fulfilled for the United States, nearly met for the average OECD economy, but missed by the EU and by Japan.

JEL classification: J10, J26, O30, O41.

Keywords: Demographic Change, Rising Life Expectancy, Pension Reforms, Long-Run Economic Growth, R&D, Innovation.

*An earlier version of this paper circulated with the title "On the long-run growth effect of raising the retirement age".

1 Introduction

Rich countries have been facing unprecedented increases in life expectancy over the past decades. For example, life expectancy in the United States increased from about 69 years in the 1950s to about 79 years in 2020, while it increased by even more in countries such as France and Germany, from about 67 years in the 1950s to more than 80 years in 2020 (United Nations, 2019). This development undoubtedly raises individual well-being (Kuhn and Prettner, 2016; Baldanzi et al., 2019). However, it also comes with certain economic concerns. If people live longer for a constant retirement age, the resulting increase in economic dependency could lead to a reduction in economic growth and pose a threat to the sustainability of social security systems and pension funds (Gruber and Wise, 1998; World Economic Forum, 2004; The Economist, 2009, 2011; Bloom et al., 2010). The extent to which this is true, however, also depends on the extent to which individuals change their savings behavior in response to increasing longevity and on the extent to which retirement policies are adjusted to cope with demographic change (Bloom et al., 2007, 2010).

The economic effects of changing life expectancy and changing retirement policies have been analyzed, for example, by Futagami and Nakajima (2001), Heijdra and Romp (2009), Bloom et al. (2007, 2014), Heijdra and Mierau (2011), and Prettner and Canning (2014). These papers either assume a partial equilibrium perspective in which the interest rate and economic growth do not react to individual decisions, or they are based on models in which growth is driven by physical capital accumulation either with decreasing returns to capital accumulation a la Solow (1956), Cass (1965), Koopmans (1965), and Diamond (1965) or with constant returns to capital accumulation due to learning-by-doing spillovers a la Romer (1986) and Rebelo (1991). However, long-run economic growth in rich countries is mainly driven by purposeful research and development (R&D) investments (Romer, 1990; Aghion and Howitt, 1992; Jones, 1995; Kortum, 1997). To analyze the effects of rising life expectancy and changing the retirement age for these economies, we therefore integrate a demographic structure of overlapping generations in the vein of Blanchard (1985) into the endogenous R&D-based growth model of Romer (1990) with a given retirement age.

In a first step, we show that increasing the retirement age raises economic growth and the equilibrium interest rate because the positive growth effect of the larger workforce implied by a longer working life overcompensates for the negative growth effect of reduced savings. Thus, raising the retirement age might be an accurate policy response to the phenomenon of secular stagnation in which economic growth is sluggish and the equilibrium interest rate is stuck below zero (Eggertsson et al., 2019, 2020).

The savings channel – and the savings channel only – also features in Futagami and Nakajima (2001) and Heijdra and Mierau (2011) who find that an increase in the retirement age leads to a *reduction* of capital accumulation and therefore to a *reduction* of economic growth in a Romer (1986) model in which growth is driven by capital accumulation with learning-by-doing spillovers. While their results are highly relevant for economies with exogenous technological progress (e.g., mostly small economies adopting technologies from abroad), our results imply the opposite effect in countries that drive the world-wide technological frontier such as Germany, Japan, and the United States. The opposing results are also interesting from a theoretical point of view: It is often argued that results based on endogenous growth models with learning-by-doing spillovers (Romer, 1986) are very similar to the results based on R&D-driven growth models (Romer, 1990). Our contribution shows that this is not the case when analyzing the implications of an increase in the retirement age. Thus, from a policy perspective, it is important to keep the underlying structure of the economy into account when considering retirement policies.

In a second step, we show that the extent to which economic growth changes in response to rising life expectancy depends on the accompanying pension policies. This is because as long as the boost to savings that is brought about by an increase in longevity is not very strong, the increase in economic dependency that comes with greater longevity *for a given retirement age* will typically lead to a reduction in R&D activity and economic growth. We show that an increase in the retirement age in proportion to an increase in life expectancy is sufficient for an increase in longevity to stimulate economic growth. We also provide a necessary and sufficient condition for a positive growth impulse, requiring the elasticity of the retirement age with respect to the increase in longevity to be sufficiently large. Our analysis concludes with a set of numerical examples, studying how the United States, the European Union economy, Japan, and the average OECD economy have fared in respect to the rise in longevity over the time span 2000–2017. We find that only the United States have clearly benefited in terms of higher economic growth, whereas the growth stimulus from the longevity increase was neutral for the OECD and negative for the European Union and Japan. Our numerical results also suggest that a longevity-driven boost to savings is much weaker than the effects that run through changes in labor participation.

The paper is organized as follows. In Section 2 we set up an overlapping generations version of the R&D-based endogenous growth model of Romer (1990) with a fixed retirement age. In Section 3 we derive our main results and discuss their relevance for actual retirement policies carried out in different countries. Finally, in Section 4 we draw our conclusions.

2 The model

2.1 Household side

Consider an economy in which individuals enter the labor market as adults at time t_0 and maximize their remaining discounted stream of lifetime utility given by

$$U = \int_{t_0}^{\infty} \log(c) e^{-(\rho+\mu)(t-t_0)} dt, \tag{1}$$

where c is instantaneous consumption, ρ is the pure rate of time preference, and μ represents the mortality rate that augments the rate of time preference because the risk of death constitutes a further reason to consume earlier in life rather than later. Individuals earn non-capital income w (wages and lump-sum redistributions of profits from intermediate goods producers) as long as they are not retired. Suppressing time arguments and following Yaari (1965) in assuming that individuals save in terms of fair annuities that insure against the risk of dying with positive capital holdings, the flow budget constraint reads

$$\dot{k} = \chi w + (\mu + r)k - c, \tag{2}$$

where k denotes the individual capital stock and χ is an indicator function with value 1 when working and 0 when retired (Bloom et al., 2007; Prettner and Canning, 2014). The first term in the flow budget constraint relates to income earned on the labor market and from receiving the lump-sum redistributions of profits (Kuhn and Prettner, 2016). This term becomes zero once an individual retires. The second term refers to the interest earnings on capital holdings (rk), which are augmented by the redistribution of capital from people who die to those who survive via the annuity market (μk). If individuals have a higher income than their consumption expenditures at a given instant, their capital stock accumulates ($\dot{k} > 0$).

Solving the intertemporal maximization problem as represented by Equations (1) and (2) leads to the standard Euler equation

$$\dot{c} = (r - \rho)c \tag{3}$$

stating that consumption expenditure growth depends positively on the difference between the interest rate and the rate of time preference.

The lifetime budget constraint is

$$\int_{t_0}^{\infty} e^{-(\mu+r)(t-t_0)} c(t_0, t) dt = \int_{t_0}^{t_0+R} e^{-(\mu+r)(t-t_0)} w(t_0, t) dt,$$

where lifetime consumption expenditures (the left-hand side) have to equal lifetime income (the right-hand side). In this expression, the working life span is denoted by R such that the age at retirement is given by $t_0 + R$ in the upper bound of the integral on the right-hand-side that represents lifetime income. For a constant age at labor market entry t_0, an increase in R is tantamount to an increase in the retirement age.

Denoting the aggregate capital stock by K and aggregate consumption expenditures by C, we have the following definitions to derive the corresponding variables (see for example Blanchard, 1985; Prettner, 2013; Heijdra, 2017, chapter 15):

$$K(t) \equiv \int_{-\infty}^{t} k(t_0, t)N(t_0, t)dt_0, \tag{4}$$

$$C(t) \equiv \int_{-\infty}^{t} c(t_0, t)N(t_0, t)dt_0, \tag{5}$$

where $N(t_0, t)$ denotes the size of the cohort entering the labor market at time t_0 as of date t, while $k(t_0, t)$ and $c(t_0, t)$ are the capital holdings and consumption levels of the members of this cohort at time t, respectively. To capture the case of a stationary population, which is reasonably close to the situation in many rich countries, we assume that the birth rate equals the death rate. In this case, the flow of labor market entrants is $N(t, t) = \mu N(t)$, where $N(t) = \int_{-\infty}^{t} N(t_0, t)dt_0 \equiv N$ represents the adult population size and $L(t) = \int_{t-R}^{t} N(t_0, t)dt_0$ is the labor force. Note that, in our setting, i) each adult cohort is of size $\mu N e^{\mu(t_0 - t)}$ at a certain date $t > t_0$, ii) the cohort fertility rate stays constant for a changing mortality rate μ such that the fertility decisions of individuals do not change for changing parameters, and iii) a change in the mortality rate does not change the population size such that the scale effect in the Romer (1990) framework is neutralized with respect to the overall population size.[1]

Taking into account our demographic structure and using the stated aggregation rules leads to the following dynamic equations for the aggregate capital stock and for aggregate consumption

$$\dot{K} = rK + W - C, \tag{6}$$

$$\frac{\dot{C}}{C} = r - \rho - \mu(\rho + \mu)\frac{K}{C}, \tag{7}$$

where W refers to aggregate non-capital income.

The resource constraint states that aggregate production Y is either consumed or invested in physical capital such that the goods market clearing condition

$$\dot{K} = Y - C \tag{8}$$

[1] Assuming a growing population as in Buiter (1988) together with a semi-endogenous growth framework as in Jones (1995) would imply similar effects as the ones that we find during the transition to the long-run balanced growth path.

is fulfilled. Next we turn to the description of the production side of the economy. Since the production side closely follows the R&D-based endogenous growth literature, we only state the most important equations that we need for the further analysis of changing the retirement age and changing life expectancy.

2.2 Production side

There are two types of workers in the economy. The first type (L_Y) is employed in the final goods sector to assemble the consumption aggregate. The second type (L_A) refers to scientists in the R&D sector who develop the new technologies ("blueprints" for machines or simply "ideas") that drive productivity growth in knowledge-based economies (see, for example, Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). To produce the consumption aggregate, the representative firm in the final goods sector combines workers and machines according to the production technology

$$Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha di, \tag{9}$$

where A is the stock of technology in the country, x_i is the quantity of a specific machine i used in final goods production, and $\alpha \in (0,1)$ is the elasticity of final output with respect to machines. Taking the final good as the numéraire, profit maximization of final goods producing firms together with the assumption of perfect competition in the final goods market imply that the wage rate for workers, w_Y, and the price of machines, p_i, are given by

$$w_Y = (1-\alpha)\frac{Y}{L_Y}, \qquad p_i = \alpha L_Y^{1-\alpha} x_i^{\alpha-1}. \tag{10}$$

Each intermediate firm produces one of the differentiated machines such that there is monopolistic competition in the vein of Dixit and Stiglitz (1977). After a firm has purchased a blueprint from the R&D sector, it has access to a production technology that allows them to convert one unit of capital k into one machine x such that $k_i = x_i$ for all firms i. Thus, operating profits can be written as

$$\pi_i = p_i k_i - r k_i = \alpha L_Y^{1-\alpha} k_i^\alpha - r k_i. \tag{11}$$

Profit maximization of firm i yields the optimal pricing policy $p_i = r/\alpha$, where $1/\alpha$ is the markup over marginal cost (Dixit and Stiglitz, 1977). As a consequence, the aggregate capital stock is equal to the total quantity of intermediates, i.e., $K = Ax$. Using this, the aggregate production function [Equation (9)] becomes $Y = (AL_Y)^{1-\alpha} K^\alpha$ and production per capital unit can be written as a function of the interest rate and the elasticity of final output with respect to machines $r = \alpha p = \alpha^2 Y/K \Rightarrow Y/K = r/\alpha^2$.

The R&D sector employs scientists to discover new technologies (Romer, 1990).

Depending on the productivity of scientists, λ, and their employment level, L_A, the stock of blueprints evolves according to

$$\dot{A} = \lambda A L_A. \tag{12}$$

R&D firms maximize their profits $\pi_A = p_A \lambda A L_A - w_A L_A$, with p_A representing the price of a blueprint, by choosing the employment level, L_A. The first-order condition of this profit maximization problem pins down wages in the research sector as $w_A = p_A \lambda A$. Due to perfect labor mobility, wages of workers in the final goods sector and wages of scientists equalize at the labor market equilibrium such that

$$w_A = p_A \lambda A = (1-\alpha)\frac{Y}{L_Y} = w_Y. \tag{13}$$

Firms in the R&D sector charge a price for the blueprint they produce that is equal to the present value of operating profits in the intermediate goods sector. The reason is that there is always a potential entrant who is willing to outbid a lower price. Consequently,

$$p_A = \int_t^{\infty} e^{-\Omega(\tau)} \pi \, d\tau$$

holds in equilibrium, where $\Omega(\tau) = \int_t^{\tau} r(s) \, ds$ denotes the compound interest between t and τ. Via the Leibniz rule and the fact that prices of blueprints do not change along a balanced growth path (BGP), we obtain the long-run equilibrium price of a blueprint as

$$p_A = \frac{\pi}{r}.$$

Next, by using Equation (11), we get operating profits as $\pi = (1-\alpha)\alpha Y/A$ such that the price of blueprints becomes $p_A = (1-\alpha)\alpha Y/(rA)$. Using the labor market clearing condition $L = L_A + L_Y$, we can then determine the quantity of labor employed in the final goods sector and in the R&D sector by using Equation (13) as

$$L_Y = \frac{r}{\alpha\lambda}, \qquad L_A = L - \frac{r}{\alpha\lambda}. \tag{14}$$

This endogenous division of labor determines the flow of new technologies in the R&D sector. Inserting Equation (14) into Equation (12) leads to the evolution of technology as

$$\dot{A} \;=\; \max\left\{\lambda A L - \frac{rA}{\alpha} \;,\; 0\right\}. \tag{15}$$

Now we have all the necessary ingredients to solve for the long-run BGP and to assess the effects of changing life expectancy and a changing retirement age on economic growth.

3 The impact of rising longevity and changing retirement policies on long-run growth

Along a BGP, we know that the growth rates of technology, capital, and consumption coincide such that $\dot{A}/A = \dot{C}/C = \dot{K}/K = g$. Collecting Equations (7), (8), (15), recalling that labor supply is given by $L(t) = \int_{t-R}^{t} N(t_0, t)\ dt_0$, and utilizing the definition $C/K \equiv \xi$, we derive the following three-dimensional system describing our model economy along the BGP

$$g = \frac{r}{\alpha^2} - \xi, \tag{16}$$

$$g = r - \rho - \mu(\rho + \mu)\frac{1}{\xi}, \tag{17}$$

$$g = \lambda \int_{t-R}^{t} N(t_0, t)\ dt_0 - \frac{r}{\alpha}. \tag{18}$$

In this system, the endogenous variables are the interest rate (r), the consumption-to-capital ratio (ξ), and the long-run economic growth rate (g). Since this system cannot be solved explicitly, we turn to implicit comparative statics to prove the analytical results in Propositions 1 and 2. We focus on two sets of insights: In a first step, we consider the balanced-growth impact of an isolated change in the retirement age. In a second step, we consider the balanced-growth impact of a change in longevity and study how this depends on potential adjustments in the retirement age.

Proposition 1. *In the endogenous growth framework of Romer (1990) with overlapping generations and retirement, an increase in the retirement age (a rise in R) leads to*

(i) an increase in the interest rate (r) and

(ii) an increase in the long-run economic growth rate (g).

Proof. Noting that $\int_{t-R}^{t} N(t_0, t)\ dt_0 = N\left(1 - e^{-\mu R}\right)$, we rewrite the system (16)-(18) as

$$W(\xi, g, r) := \frac{r}{\alpha^2} - \xi - g = 0, \tag{19}$$

$$X(\xi, g, r) := r - \rho - \mu(\rho + \mu)\frac{1}{\xi} - g = 0, \tag{20}$$

$$Y(\xi, g, r) := \lambda N\left(1 - e^{-\mu R}\right) - \frac{r}{\alpha} - g = 0. \tag{21}$$

Applying the implicit function theorem and Cramer's rule, we obtain the following comparative statics

$$\frac{dg}{dR} = \frac{\lambda\mu N e^{-\mu R}\left[\alpha^2\xi^2+\mu(\mu+\rho)\right]}{(1+\alpha)\left[\alpha\xi^2+\mu(\mu+\rho)\right]} > 0, \tag{22}$$

$$\frac{dr}{dR} = \frac{\alpha^2\lambda\mu N e^{-\mu R}\left[\mu(\mu+\rho)+\xi^2\right]}{(1+\alpha)\left[\alpha\xi^2+\mu(\mu+\rho)\right]} > 0. \tag{23}$$

□

Hence, in contrast to Futagami and Nakajima (2001) and Heijdra and Mierau (2011), who base their analysis on a Romer (1986) framework in which growth is driven by physical capital accumulation via learning-by-doing spillovers, an increase in the retirement age unambiguously raises economic growth in a Romer (1990) setting. The intuition for this result is that a rise in the retirement age implies an increase in the labor force and therefore it raises the number of scientists that are available for the production of blueprints in the R&D sector. While there is also a reduction in individual savings due to the longer working life (as in Futagami and Nakajima, 2001; Heijdra and Mierau, 2011), the associated negative growth effect is overcompensated by the positive effect of the larger labor force.

The difference in the results suggests that in economies in which growth is mainly driven by purposeful R&D investments (such as Germany, Japan, and the United States), an increase in the retirement age will indeed lead to a rise in the long-run economic growth rate. However, in economies, in which growth is mainly driven by physical capital accumulation coupled with learning-by-doing spillovers (predominantly small economies that are not advancing the world-wide research frontier and adopt technologies developed abroad), a rise in the retirement age could lead to a reduction in the growth rate. Consequently, any adjustment of the retirement age should be considered in light of the underlying structure of the economy.

The results of Proposition 1 show that a rise in the retirement age might be an accurate policy response to the phenomenon of secular stagnation as described in detail by Eggertsson et al. (2019, 2020). According to these contributions we are facing a prolonged period of stagnation in many countries with sluggish economic growth and an equilibrium interest rate that is stuck below zero. Our results show that raising the retirement age increases the workforce and – at the same time – reduces savings. Both of these effects put upward pressure on the interest rate and, overall, lead to faster economic growth.

As far as the robustness of our results to the use of other R&D-based growth models as baseline framework is concerned, the following remark is in order.

Remark 1. *The impact of an increase in the retirement age would follow a similar mechanism as the one described here in both a semi-endogenous growth model following Jones (1995), Kortum (1997), or Segerström (1998) and a Schumpeterian growth* *model following Peretto (1998) or Young (1998) without the strong scale effect. However, the growth effect itself would only be transient and vanish in the long run. In other words, increasing the retirement age would have a positive level effect on per capita GDP but not on long-run growth. As the transition in these growth models is usually rather slow (see, for example, Prettner and Trimborn, 2017), even a transient growth effect of retirement policies may span a substantial time period.*

The effects of increasing life expectancy are more subtle because there are three separate and opposing channels: (i) a reduction of the generational turnover that leads to higher aggregate savings, which encourages investment in R&D and thereby fosters economic growth; (ii) a reduction of the labor supply for a given retirement age because – for a stationary population – a lower mortality rate implies a lower support ratio; and (iii) potentially an offsetting adjustment in the retirement age by policy makers.

Proposition 2. *In the endogenous growth framework of Romer (1990) with overlapping generations and retirement, an increase in life expectancy (a decrease in μ) leads to*

(i) a decrease in the interest rate (r) if and only if the retirement response satisfies

$$\frac{dR}{d\mu}\frac{\mu}{R} > -1 - \frac{\xi(\rho+2\mu)}{R\lambda N e^{-\mu R}\left[\mu(\rho+\mu)+\xi^2\right]}, \tag{24}$$

(ii) an increase in the long-run economic growth rate (g) if and only if the retirement response satisfies

$$\frac{dR}{d\mu}\frac{\mu}{R} < -1 + \frac{\alpha\xi(\rho+2\mu)}{R\lambda N e^{-\mu R}\left[\mu(\rho+\mu)+\alpha^2\xi^2\right]}. \tag{25}$$

Proof. Applying the implicit function theorem and Cramer's rule to the system of equations (19)-(21), we obtain the following comparative statics

$$\frac{dg}{d\mu} = \frac{e^{-\mu R}\left\{\lambda N R\left[\alpha^2\xi^2+\mu(\mu+\rho)\right]-\alpha\xi(2\mu+\rho)e^{\mu R}\right\}}{(\alpha+1)\left[\alpha\xi^2+\mu(\mu+\rho)\right]} \lesseqqgtr 0,$$

$$\frac{dr}{d\mu} = \frac{\alpha^2\left\{\xi(2\mu+\rho)+\lambda N R e^{-\mu R}\left[\mu(\mu+\rho)+\xi^2\right]\right\}}{(\alpha+1)\left[\alpha\xi^2+\mu(\mu+\rho)\right]} > 0.$$

Combining these expressions together with (22) and (23), we obtain the forms

$$\frac{dg}{d\mu}+\frac{dg}{dR}\frac{dR}{d\mu}$$

and

$$\frac{dr}{d\mu}+\frac{dr}{dR}\frac{dR}{d\mu}.$$

Rearranging these expressions provides the conditions reported in the proposition.

□

Remark 2. *We have cast our analysis directly in terms of the mortality rate* μ. *In the Blanchard (1985) setting life expectancy is given by the identity* $LE \equiv \mu^{-1}$. *Thus, we obtain the relationship*

$$\frac{d\mu}{dLE} = -\frac{\mu}{LE}$$

by which to multiply all relevant derivatives to obtain the corresponding expression in terms of life expectancy. Furthermore, note that

$$\frac{dR}{dLE}\frac{LE}{R} = \frac{dR}{d\mu}\frac{d\mu}{dLE}\frac{R}{LE} = -\frac{dR}{d\mu}\frac{\mu}{R}$$

implying that the two conditions in (24) and (25) can be written in terms of life expectancy by simply reversing the sign of the right-hand-side terms.

To establish a benchmark, consider that the retirement age is not adjusted to a change in longevity, i.e., $dR/d\mu \equiv 0$. It is immediately verified for this case that $dr/d\mu > 0$, implying that the interest rate decreases in response to an increase in life expectancy, reflecting a boost in savings. By contrast, we have $dg/d\mu \lesseqgtr 0$, implying an ambiguous effect of longevity on economic growth. This is because it is not clear whether the shift of resources into the R&D sector that is triggered by the decline in the interest rate (Prettner et al., 2013; Kuhn and Prettner, 2016) compensates for the reduction in labour force participation. The proposition then provides conditions on the elasticity of the retirement response to an increase in longevity for the overall impact on economic growth to be positive. Specifically, an increase in the retirement age in response to an increase in longevity (decline in mortality) is required for a positive impact on economic growth whenever the right-hand-side of the inequality in (25) is negative, which is true if the savings response is relatively weak. Notably, however, the retirement age may be lowered, i.e.,

$$-\frac{dR}{d\mu}\frac{\mu}{R} < 0,$$

in cases in which the increase in longevity translates into a relatively strong savings response (as may be true for very low levels of the retirement age). Furthermore, the condition shows that a proportional adaptation of the retirement age to longevity, as implied by the unit elasticity

$$-\frac{dR}{d\mu}\frac{\mu}{R} = 1$$

always guarantees a positive impact of a longevity increase on balanced growth. Finally, we see from (24) that longevity improvements lead to a decline in the interest rate as long as retirement is not boosted by too much. Again, this is always true for a proportional adaptation.

The ambiguous effect of an increase in life expectancy on economic growth in case of a fixed retirement age begs the questions i) as to how much different economies need to adjust their retirement age in order to secure a positive impact of rising longevity on economic growth and ii) as to whether actual changes to the retirement age are adequate in this regard. In the following, we provide a numerical assessment of how four economies – the United States, the EU, Japan, and the OECD (average) economy – have fared in respect to these questions. To obtain a long-term assessment and to balance out short-term fluctuations, we calibrate the economies to reflect the average growth rate over the time frame 2000–2017, dropping the years 2008 and 2009 due to the strong distortions during the financial crisis, and calculate the threshold value

$$\Psi := -1 + \frac{\alpha\xi(\rho + 2\mu)}{R\lambda N e^{-\mu R}\left[\mu(\rho + \mu) + \alpha^2\xi^2\right]}$$

on this basis. We then calculate the elasticity of the change in the retirement age in response to the change in life expectancy over the time frame 2000–2017 and compare it against the threshold in order to arrive at an assessment of the impact of longevity increases on economic growth.

Successively substituting for r and ξ in the system (16)–(18) and subsequently reinserting, we can solve for the closed form solution

$$g^* = \frac{\left\{\begin{array}{c}(1+\alpha)\,\lambda N\left(1-e^{-\mu R}\right), \\ -\rho - \sqrt{\left[(1-\alpha)\,\lambda N\left(1-e^{-\mu R}\right)+\rho\right]^2 + 4\alpha\mu\,(\mu+\rho)}\end{array}\right\}}{2\,(1+\alpha)}, \tag{26}$$

$$r^* = \alpha\lambda N\left(1-e^{-\mu R}\right) - \alpha g^*, \tag{27}$$

$$\xi^* = \frac{r^*}{\alpha^2} - g^* = \frac{\lambda N\left(1-e^{-\mu R}\right)}{\alpha} - \frac{1+\alpha}{\alpha}g^*. \tag{28}$$

For all countries, we set $\alpha = 0.33$ in line with Jones (1995) and Acemoglu (2009) and a time preference rate $\rho = 0.025$ that is within a reasonable range of values (cf. Warner and Pleeter, 2001; Grossmann et al., 2013a,b). We take the economy-specific growth rate g and life expectancy $LE = \mu^{-1}$ from the World Development Indicators (World Bank, 2019) and calculate the average effective retirement age R from OECD (2019) data.[2] Based on this, we employ Equation (26) to calibrate the value of λN. Using

[2]According to OECD (2019) "the average effective age of retirement is calculated as a weighted average of (net) withdrawals from the labour market at different ages over a 5-year period for workers initially aged 40 and over. In order to abstract from compositional effects in the age structure of the population, labour force withdrawals are estimated based on changes in labour force participation rates rather than labour force levels." OECD (2019) reports the average effective retirement ages by gender. To arrive at a general population average, we weigh the gender-specific retirement ages with the gender-shares in the total labor force, as reported in World Bank (2019).

this value in Equation (28) to determine ξ^*, we derive the threshold Ψ to which we compare the elasticity

$$\epsilon := \frac{(R_{2010-2017} - R_{2000-2007})/R_{2000-2007}}{(LE_{2010-2017} - LE_{2000-2007})/LE_{2000-2007}}$$

that we calculate from the data. When calculating the elasticity, we average the values of retirement and life expectancy over the time spans 2000–2007 and 2010–2017, respectively. Table 1 summarizes our findings.

Variable	United States	EU	Japan	OECD
avg. growth rate 2000–2017 (in %)	2.16	1.75	1.48	1.60
avg. life expectancy 2000–2017	77.96	79.28	82.62	78.90
avg. retirement age 2000–2017	64.84	61.55	68.53	63.10
threshold (Ψ)	0.944	0.951	0.946	0.947
elasticity (ϵ)	1.260	0.582	0.484	0.900
net gain in avg. growth 2000–2017 vs. 2000–2010 (in % points)	0.013	-0.042	-0.02	-0.003

Table 1: Comparison between the threshold (Ψ) and the elasticity (ϵ). Source: authors' own calculations based on World Bank (2019) (growth rates, life expectancy, gender shares in total labor force) and OECD (2019) (average effective retirement age).

The four economies exhibit some variation across growth rates, with the United States experiencing average growth rates in excess of 2 percent over the time frame 2000–2017 as opposed to some 1.5 percent only in Japan. By contrast, Japan leads in terms of life expectancy with around 83.5 years, while the United States is lagging with some 78 years. With 68.5 years the Japanese retire very late and experience only around 15 years in retirement, whereas EU citizens live more than 17.5 years in retirement. The threshold value for the necessary increase in the retirement age is rather close to one for all economies, which reflects a modest and rather similar marginal savings response to an increase in longevity. This, in turn, suggests that for all economies there is relatively little leeway to remain below an increase in the retirement age that would be in lockstep with the increase in longevity unless they were willing to forgo economic growth.

As it turns out, however, there is considerable variation in the elasticities of the actual retirement age with respect to longevity change, ranging from 1.26 in the United States, implying an overcompensation of the increase in longevity, to 0.48 in Japan. According to our results, only the United States would have experienced a positive growth stimulus from the increase in longevity, whereas Japan and the EU economy would have suffered a loss. Notably, the OECD average economy lies very close to the threshold, implying that the longevity increase was almost neutral. We conclude by noting that the differentials in the simulated average growth rates 2000–2007 as opposed to 2010–2017 are rather small. They range from a 0.013 percentage point gain in the United States to a 0.042 percentage point loss in the EU. This suggests that the impact on economic growth may be of secondary importance in the determination of retirement policies.

4 Conclusions

We showed that a rise in the retirement age implies faster long-run economic growth in modern knowledge-based economies and that the growth effects of increasing life expectancy depend on the underlying retirement policies. If the retirement age is left constant, an increase in life expectancy is likely to reduce economic growth. By contrast, if the retirement age rises in lockstep with life expectancy, this is sufficient for economic growth to be boosted by an increase in life expectancy. We provide a more specific threshold requirement for the increase in the retirement age that is necessary to preserve a positive growth stimulus and show numerically that this criterion is overachieved by the United States, (nearly) met for the OECD average, and not attained by the EU and Japan. The growth impact is of relatively modest magnitude, however.

Overall, our results differ from those based on models in which growth is mainly driven by physical capital accumulation coupled with learning-by-doing spillovers. This modeling reflects small economies that are not advancing the world-wide research frontier and mainly adopt technologies developed abroad. In these economies, a rise in the retirement age is prone to depress the long-run economic growth rate (Futagami and Nakajima, 2001; Heijdra and Mierau, 2011). Overall, our results therefore suggest that keeping the underlying structure of the economy in mind is particularly important when conducting pension policies.

The policy advices emanating from our R&D-based endogenous growth model with demography and retirement would be i) to raise the retirement age in the face of secular stagnation because this boosts economic growth and puts upward pressure on the interest rate; and ii) to couple "on average" the retirement age to life expectancy. However, we are well aware that crucial differences between different types of labor are present in reality. For employees in the R&D sector it might easily be possible and even desirable to extend the working age, whereas workers in the production sector may struggle, e.g., due to health issues. Consequently, retirement may well need to be designed in a flexible way such that an increase in the retirement age is possible for those who are still able and willing to work, while there are options for earlier retirement (potentially coupled with actuarially fair reductions in pension entitlements) in physically demanding occupations and for those with health problems.[3]

[3] See e.g. Kuhn et al. (2015) for a theoretical analysis under which conditions early retirement may be attractive on welfare grounds.

A second qualifier is that we are currently observing breathtaking advances in automation technologies that are replacing workers (Acemoglu and Restrepo, 2018b; Prettner and Strulik, 2019; Prettner, 2019). These advances are particularly pronounced in countries that are subject to fast population aging (Abeliansky and Prettner, 2017; Acemoglu and Restrepo, 2017, 2018a). While it remains to be seen whether automation could help to avert the negative effects of a declining workforce, this possibility is worth to be considered in the context of future social security and pension policies.

Appendix

A Optimal consumption and retirement

The control variable is consumption c. The current value Hamiltonian is given by

$$H = \log(c) + \phi \left[\chi w + (\mu + r)k - c \right].$$

The first order conditions (FOCs) are

$$\frac{1}{c} = \phi,$$
$$(\mu + r)\phi = (\rho + \mu)\phi - \dot{\phi}.$$

From the first FOC we get $-\dot{c}/c^2 = \dot{\phi}$ such that the second FOC implies the consumption Euler equation (3)

$$\frac{\dot{c}}{c} = (r - \rho)c. \tag{A.1}$$

B Aggregating over cohorts

Due to our demographic structure, the aggregation rules to calculate aggregate vonsumption and aggregate capital are given by

$$C(t) = \mu N \int_{-\infty}^{t} c(t_0, t) e^{\mu(t_0 - t)} dt_0, \tag{B.1}$$

$$K(t) = \mu N \int_{-\infty}^{t} k(t_0, t) e^{\mu(t_0 - t)} dt_0. \tag{B.2}$$

Differentiating (B.1) and (B.2) with respect to time yields

$$\begin{aligned}
\dot{C}(t) &= \mu N\left[\int_{-\infty}^{t}\dot{c}(t_0,t)e^{\mu(t_0-t)}dt_0-\mu\int_{-\infty}^{t}c(t_0,t)e^{\mu(t_0-t)}dt_0\right]+\mu Nc(t,t)\\
&= \mu Nc(t,t)-\mu C(t)+\mu N\int_{-\infty}^{t}\dot{c}(t_0,t)e^{-\mu(t-t_0)}dt_0 \qquad \text{(B.3)}\\
\dot{K}(t) &= \mu N\left[\int_{-\infty}^{t}\dot{k}(t_0,t)e^{\mu(t_0-t)}dt_0-\mu\int_{-\infty}^{t}k(t_0,t)e^{\mu(t_0-t)}dt_0\right]+\mu Nk(t,t)\\
&= \mu Nk(t,t)-\mu K(t)+\mu N\int_{-\infty}^{t}\dot{k}(t_0,t)e^{-\mu(t-t_0)}dt_0. \qquad \text{(B.4)}
\end{aligned}$$

Newborns do not own any capital because we abstract from bequests, i.e., $k(t,t)=0$. From equation (2) it then follows that

$$\begin{aligned}
\dot{K}(t) &= -\mu K(t)+\mu N\int_{-\infty}^{t}\left[\chi(t_0,t)w(t)+(\mu+r)k(t_0,t)-c(t_0,t)\right]e^{-\mu(t-t_0)}dt_0\\
&= rK(t)-C(t)+W(t),
\end{aligned}$$

which is the law of motion for aggregate capital with W being aggregate non-capital income defined as $\mu N\int_{-\infty}^{t}\chi(t_0,t)w(t)dt_0e^{-\mu(t-t_0)}$.

Reformulating an agent's optimization problem subject to the lifetime budget constraint as in Prettner and Canning (2014), we have

$$\begin{aligned}
\max_{c(t_0,\tau)} \quad & U=\int_{t}^{\infty}e^{(\rho+\mu)(t-\tau)}\log[c(t_0,\tau)]d\tau\\
s.t. \quad & k(t_0,t)+\int_{t}^{R+t}w(\tau)e^{-D^A(t,\tau)}d\tau=\int_{t}^{\infty}c(t_0,\tau)e^{-D^A(t,\tau)}d\tau,
\end{aligned} \qquad \text{(B.5)}$$

where the discount factor is $D^A(t,\tau)=\int_t^\tau[r(s)+\mu]ds$. The FOC is

$$\frac{1}{c(t_0,\tau)}e^{(\rho+\mu)(t-\tau)}=\mu(t)e^{-D^A(t,\tau)}.$$

For the period $\tau=t$ this implies that

$$c(t_0,t)=\frac{1}{\mu(t)}.$$

Therefore we can write

$$\begin{aligned}
\frac{1}{c(t_0,\tau)}e^{(\rho+\mu)(t-\tau)} &= \frac{1}{c(t_0,t)}e^{-D^A(t,\tau)},\\
c(t_0,t)e^{(\rho+\mu)(t-\tau)} &= c(t_0,\tau)e^{-D^A(t,\tau)}.
\end{aligned}$$

Integrating over time and using (B.5) provides

$$\begin{aligned}\int_t^\infty c(t_0,t)e^{(\rho+\mu)(t-\tau)}d\tau &= \int_t^\infty c(t_0,\tau)e^{-D^A(t,\tau)}d\tau,\\ \frac{c(t_0,t)}{\rho+\mu}\left[-e^{(\rho+\mu)(t-\tau)}\right]_t^\infty &= k(t_0,t)+\int_t^T w(\tau)e^{-D^A(t,\tau)}d\tau,\\ \Rightarrow c(t_0,t) &= (\rho+\mu)\left[k(t_0,t)+h(t)\right], \qquad \text{(B.6)}\end{aligned}$$

where $h=\int_t^{R+t} w(\tau)e^{-D^A(t,\tau)}d\tau$ refers to non-capital wealth, i.e., wage income plus lump-sum transfers of profits. These calculations show that optimal consumption is proportional to total wealth with a marginal propensity to consume of $\rho+\mu$ (Heijdra and van der Ploeg, 2002; Grafeneder-Weissteiner and Prettner, 2013; Prettner and Canning, 2014; Heijdra, 2017, chapter 15). Aggregate consumption is then given by

$$\begin{aligned}C(t) &\equiv \mu N\int_{-\infty}^t c(t_0,t)e^{\mu(t_0-t)}dt_0 = \mu N\int_{-\infty}^t e^{\mu(t_0-t)}(\rho+\mu)\left[k(t_0,t)+h(t)\right]dt_0\\ &= (\rho+\mu)\left[K(t)+H(t)\right]. \qquad \text{(B.7)}\end{aligned}$$

where H refers to aggregate non-capital income. Since newborns do not own capital because there are no bequests, their consumption is given by

$$c(t,t)=(\rho+\mu)h(t). \qquad \text{(B.8)}$$

Using equations (A.1), (B.3), (B.7) and (B.8), we finally get

$$\begin{aligned}\dot{C}(t) &= \mu(\rho+\mu)H(t)-\mu(\rho+\mu)\left[K(t)+H(t)\right]+\\ &\quad \mu N\int_{-\infty}^t (r-\rho)c(t_0,t)e^{-\mu(t-t_0)}dt_0\\ &= \mu(\rho+\mu)H(t)-\mu(\rho+\mu)\left[K(t)+H(t)\right]+(r-\rho)C(t)\\ \Rightarrow \frac{\dot{C}(t)}{C(t)} &= r-\rho+\frac{\mu(\rho+\mu)H(t)-\mu(\rho+\mu)\left[K(t)+H(t)\right]}{C(t)}\\ &= r-\rho-\mu(\rho+\mu)\frac{K(t)}{C(t)},\end{aligned}$$

which is the Euler equation for aggregate consumption.

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